

Erratum

Volume 8, Number 3 (1974), in the article "A Generalized Approach to Formal Languages" by T. S. E. Maibaum, pp. 409-439:

The Fundamental Theorem is incorrect as stated, as are some of its precursors (Theorems 18 and 19) and consequences (Theorems 20-23 and the corollary of the Fundamental Theorem in Section 11). The following counterexample to the Fundamental Theorem was communicated to me by J. Engelfriet and E. Meineche Schmidt.

Consider the normal form grammar G over the ranked alphabet Σ with $\Sigma_0 = \{a, b, S, A\}$, $\Sigma_1 = \{F\}$ and $\Sigma_2 = \{f\}$ and with the productions

$$\begin{aligned} S &\rightarrow F(A), \\ F(x) &\rightarrow f(xx), \\ A &\rightarrow a, \quad A \rightarrow b. \end{aligned}$$

The *OI* context free set generated by G is $\{faa, fab, fba, fbb\}$. Applying the construction in the proof of Theorem 18, we obtain the grammar G' with productions

$$\begin{aligned} S' &\rightarrow cF'A' \\ F' &\rightarrow cf\delta\delta \\ A' &\rightarrow a, \quad A' \rightarrow b. \end{aligned}$$

(Note that we have not indicated the appropriate subscripts and superscripts on the c 's and δ 's.) Now, $\text{YIELD}(L(G'))$ is the set $\{faa, fbb\}$. This is *not* the same as $L(G)$, as required by Theorem 18, and is thus a counterexample.

It should be emphasized, however, that the theory as presented and the theorems indicated above are in fact valid if we replace references to *OI* context free grammars and indexed grammars by references to *IO* context free grammars and *IO* macrogrammars, respectively. (Note that $\text{YIELD}(L(G'))$ above is the *IO* language generated by G .)

The error in the paper was also pointed out to me by A. Arnold and M. Dauchet. (See this Journal, Vol. 13, No. 2, pp. 223-244.)

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